

CHAPTER 2 - THE PLUS-MINUS TIME ANALYSIS METHOD

2.1 INTRODUCTION

2.1.1 Purpose of the method

The Plus-Minus time analysis method has two objectives: to establish a near-surface model of the Earth in terms of subweathering layer thicknesses and seismic wave velocities; and to compute static corrections to be applied to seismic data. The surface layers of the Earth are often made of unconsolidated deposits or soil which have low seismic velocities. For example, in Western Canada the near-surface layers are generally composed of Quaternary glacial deposits (Clague, 1991), while in South America or Africa the surface layers are made of weathered soil materials. These low velocity layers will induce recording delays of seismic reflection data. The impact of these delays on reflector structure can be quite significant. The use of seismic refracted arrivals to establish the thicknesses of these layers and their seismic wave velocities represents one of the best ways to correct for these delays.

2.1.2 Theory

The refraction analysis method used is the “plus-minus” method of Hagedoorn (1959), which includes the Plus time analysis for depth analysis and the Minus time analysis for velocity determination (van Overmeeran, 1987). The basis of the Plus-Minus time analysis method lies in the traveltimes reciprocity, i.e. the traveltimes of a seismic wave from source to receiver is equal to the traveltimes in the opposite direction if source and receiver are interchanged. The Plus time analysis uses the concept of the Delay time analysis introduced by Gardner (1939, 1967) and further developed by Hawkins (1961) and Barry (1967) (see Delay time analysis in Appendix A). The basic geometry of the Plus-Minus time analysis method is illustrated in Figure 2.1.

To be able to use the Plus time analysis, reciprocal spread data are essential so that the forward arrival spread extends at least to the position of the reverse source (S_r) and the reverse arrival spread to the position of the forward source (S_f) (Figure 2.1). The Plus-Minus time analysis window is defined by the two crossover points (forward spread (X_f) and reverse spread (X_r)), which determine the offset limit between the first layer arrivals and the second layer arrivals.

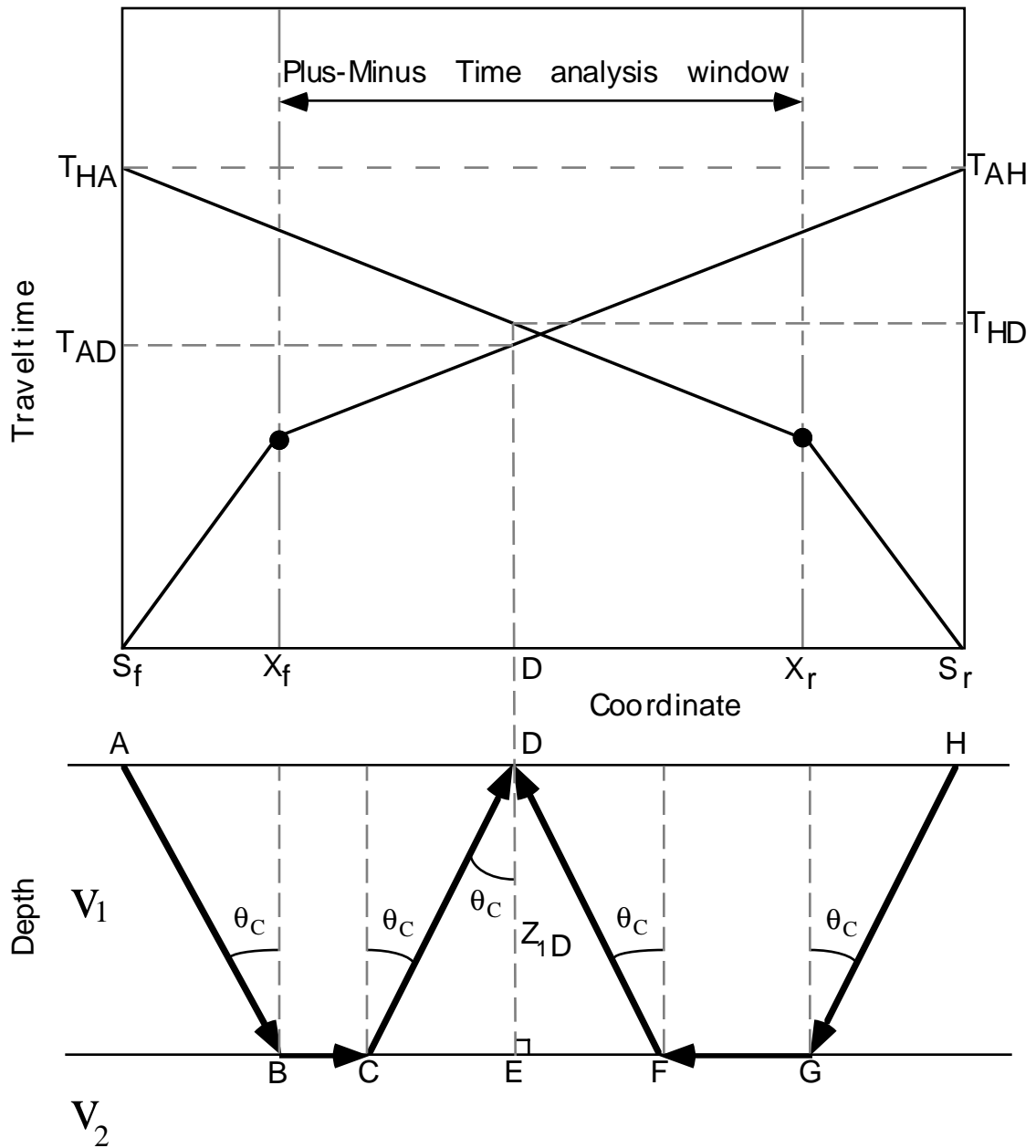


Figure 2.1. Plus time analysis according to the “plus-minus” method of Hagedoorn (1959).

The Plus time value (T^+) can be evaluated for each of the receivers inside that window. The Plus time value at a receiver (T^+D) is defined (Hagedoorn, 1959) as the sum of the traveltime at the receiver from a forward source (T_{AD}) and the traveltime at the receiver from the reverse source (T_{HD}), minus the traveltime between the two sources (T_{AH}).

$$T^+D = T_{AD} + T_{HD} - T_{AH} \quad (2.1)$$

After some substitution in Equation (2.1) and knowing that the Delay time at receiver D (δ_D) is equal to half the Plus time (T^+D) at the same receiver, a relation with the thickness (Z_1D) of the first layer below the receiver can be found.

$$Z_1D = [(T^+D) \cdot (V_1)] / 2(\cos(\theta_c)) \quad (2.2)$$

where $\theta_c = \sin^{-1}(V_1/V_2)$, V_1 and V_2 are respectively the first and second layer velocities.

The velocity of the first layer can be found using the inverse slope of the first layer arrivals (S_f to X_f and S_r to X_r). From the second layer arrivals (after X_f for the forward spread and after X_r for the reverse spread), the second layer velocity can be derived using the Minus time analysis (Figure 2.2).

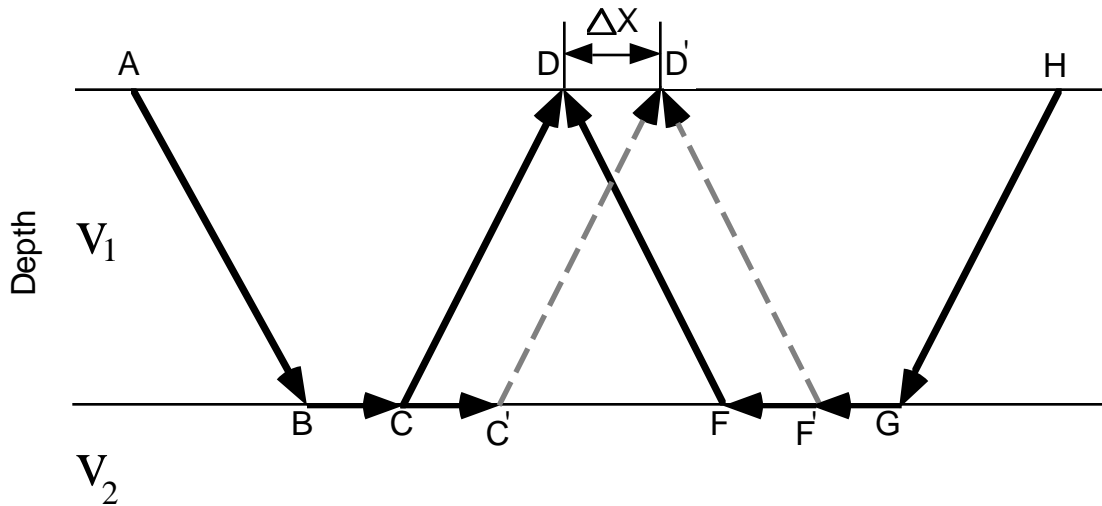


Figure 2.2. Minus time analysis according to the “plus-minus” method of Hagedoorn (1959).

The definition of the Minus time (T^-D) at a receiver is the subtraction of the traveltimes at the receiver from the reverse source (T_{HD}) of the traveltimes at the receiver from the forward source (T_{AD}), minus the traveltimes between the two sources (T_{AH}).

$$T^-D = T_{AD} - T_{HD} - T_{AH} \quad (2.3)$$

A velocity analysis of the second layer can be undertaken using the Minus time values at two receivers (D and D') with a specified separation distance (ΔX).

$$T-D' - T-D = \Delta T-D = 2(\Delta X)/V_2 \quad (2.4)$$

The velocity of the second layer (V_2) is equal to twice the inverse slope of a best fit line through the Minus time variations ($\Delta T-D$) calculated for receivers inside the Plus-Minus time analysis window.

$$V_2 = 2(\Delta X)/\Delta T-D \quad (2.5)$$

Summary

Knowing the Plus time values, the first and second layer velocities at each receiver, the thickness of the first layer below each receiver can be found according to the Delay time analysis. For more details about the Plus time and Minus time analyses, see Appendix B. The Delay time represents the time to travel from the receiver to the refractor minus the time necessary to travel the normal projection of the raypath on the refractor (Figure 2.1). From Snell's law, a relation between the Delay times and the thickness of the first layer at the receiver can be established. Finally, the link between the Delay times and the Plus times allows us to determine the thickness of the first layer below each receivers inside the Plus-Minus time analysis window (Equation 2.2). This will allow the determination of the first layer thickness all along a seismic survey line. Therefore, this analysis is restricted to a two layer case in two dimensions. The first layer velocity can be evaluated using a polynomial fit of the direct arrival or a rough estimate of the subweathering layer (first layer). The knowledge of the first layer thickness and velocity allows the determination of surface-consistent weathering static corrections, where the replacement velocity is the average refractor velocity.

2.1.3 Hardware/software requirement

The Plus-Minus time analysis software is written in the Matlab language. Matlab is a commercial package which provides a technical computing and visualization environment. A Matlab license is required to run the Plus-Minus time analysis software. Currently, arrival times and geometry are input from ProMax exported database files. The refracted arrival times are previously picked in the processing software (currently ProMax) and then input to the Plus-Minus time analysis software. The geometry files include the receiver coordinates and elevations, and the shot coordinates and elevations, as well as the uphole time (see data files section in Appendix C). The software can be run on any platform for

which Matlab is available, including UNIX, MS-Windows, and Macintosh (see PMT method implementation in Appendix C).

2.2 PROCESS

2.2.1 General flow chart

The Plus-Minus time analysis method developed for this thesis consists of seven major steps and four optional steps (Figure 2.3). As explained in the data file section (Appendix C), the refracted arrivals and the geometry files are required to properly initiate the processing. There are three final results of processing; a velocity model, a depth model, and the static corrections.

After the data are loaded, the inflection points on the refracted arrivals are identified. These are called crossover points (CVPs). The crossover points represent a change in the first arrival times between the first layer arrivals and the second layer arrivals. Once the locations of the crossover points are known for each source arrival spread, the velocities of the first layer can be calculated using a polynomial fit of the direct arrivals. The second layer velocities are calculated using the Minus time analysis with the refracted arrivals.

Simultaneously, the time delays caused by the downgoing and upgoing refracted wave raypath through the first layer can be determined using Plus time and Delay time analyses of the refracted arrivals. Once the Plus times and the two layer velocities are known, the first layer thicknesses are computed in the depth calculation processing step. Finally, from the first layer thicknesses and velocities, the static corrections which determine the effect of that layer, can be found using the static computation processing step.

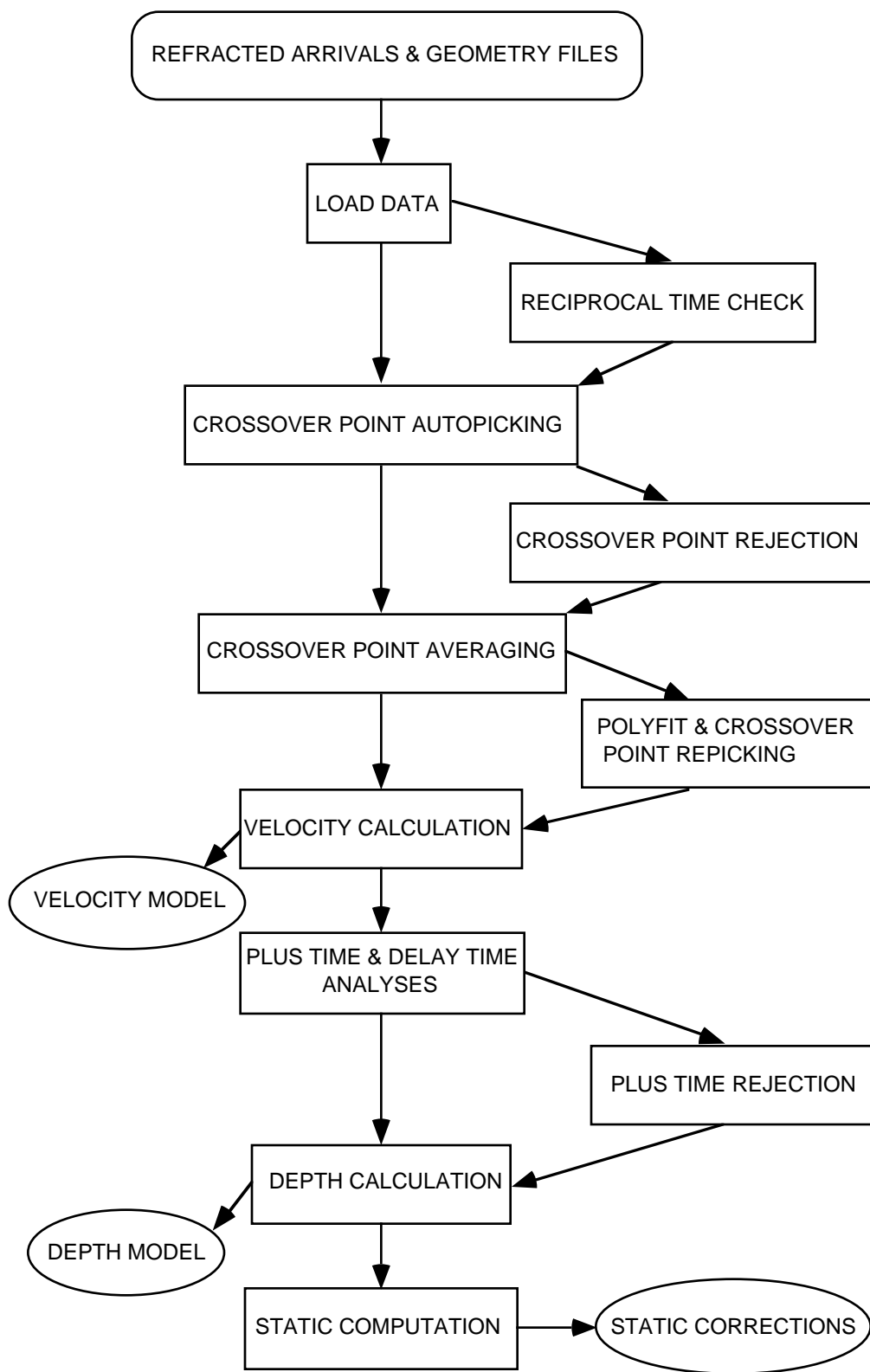


Figure 2.3. Processing flow chart

2.2.2 Reciprocal time check

As mentioned in the theory section, the basis of the Plus-Minus time analysis method lies in the traveltimes reciprocity, i.e. the traveltimes of a seismic wave between two locations in one direction is equal to the traveltimes in the opposite direction. For example, if seismic energy is sent from a shot and recorded at a certain distance by a receiver, the shot and receiver position may be switched, but the traveltimes recorded in both cases should be equal.

The first step after loading the data is to check if the traveltimes reciprocity is valid. The reciprocal time check process will determine the traveltimes difference for all possible shot pairs, i.e. the two shot spreads have overlapping receivers at the pairing shot location. Figure 2.4 shows two generalized shot gathers where the receiver spread of the i th shot overlaps the receiver spread of the j th shot. It can be noted that the traveltimes of the forward spread arrivals at the reverse shot location is equal to the traveltimes of the reverse spread arrivals at the forward shot location. The subtraction of the two traveltimes equals zero and the traveltimes reciprocity is correct.

With real data, however, the traveltimes reciprocity is generally not respected due to slight separations between shots and receivers at the same station, and also to arrival picking errors. Also, if dynamite is used, the source is often buried below the surface to variable depth. Therefore, the uphole time, which is the time taken by the seismic energy to travel back up to the surface, should be added before the reciprocity is verified. The reciprocal time check process allows the user to identify the shot pairs with reasonable reciprocal traveltimes differences and the problematic shot pairs. There are several possibilities to solve the problem of shot pairs with significant reciprocal traveltimes differences: first a review and correction of the arrival picking errors in the processing software used to create them; second the problematic shot pairs can be ignored in the subsequent process; third the reciprocal traveltimes difference of a problematic shot pair can be used to adjust the arrival traveltimes of one of the shots according to the pairing shot (bulk shift).

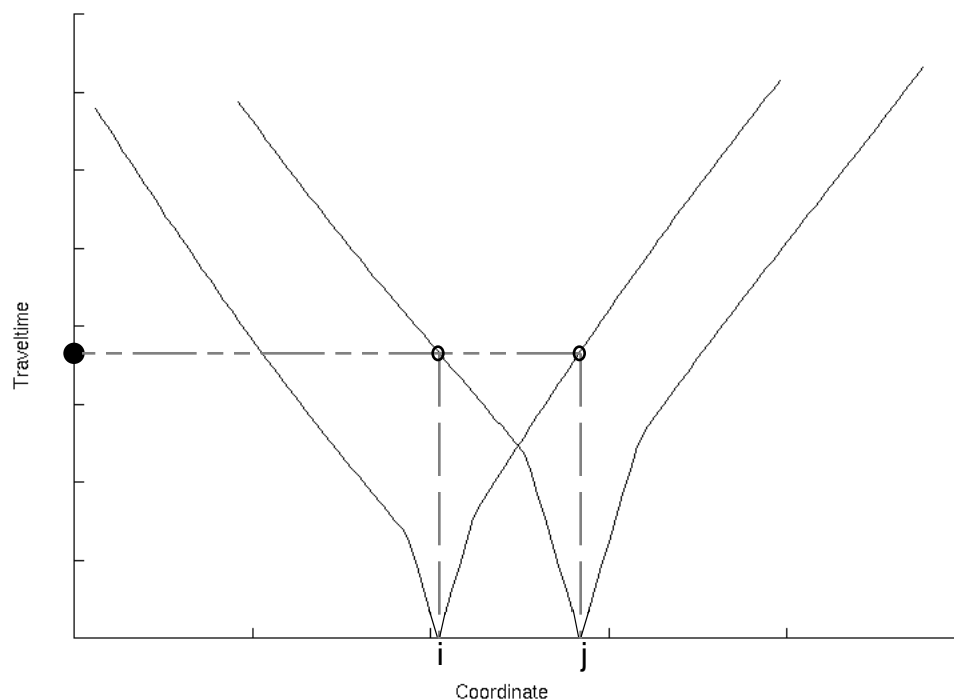


Figure 2.4. Reciprocal time check with shot gathers i and j.

2.2.3 Crossover point autopicking

Crossover point picking represents the most important step of this program. The crossover points indicate a change in the arrivals between the first and second layer. At the crossover point, the traveltime of the second layer arrivals is equal to the traveltime of the first layer arrivals. Beyond the crossover point, the refracted arrivals from the second layer are recorded before the first layer arrivals. The first layer arrivals (also called direct arrivals) are the traveltimes of a seismic wave from the source to the receivers along the surface of the first layer. The second layer arrivals (also called refracted arrivals) are the traveltimes of a seismic wave from the source to the bottom of the first layer then along the first and second layer interface and back up through the first layer to the receivers (Figure 2.5).

The position of the crossover points on each shot spread is found using the traveltime difference analysis developed by Lawton (1989) and the branch point analysis of Wang and Cheadle (1995). The traveltime difference analysis consists of subtracting two adjacent shot arrival traveltimes at common receiver locations (Figure 2.6). The traveltime differences (δt) remain constant when the arrivals from the two shots involve the same layer. The crossover point is located at the limit between a constant traveltime difference

and an increasing or decreasing traveltime difference. For example, if we subtract the traveltime of two overlapping shot gathers (Figure 2.6), it can be noticed that the traveltime difference on the left of shot i location is constant until it reaches the left crossover point of the shot gather i and then the traveltime difference increases. On the other side (right of shot j location), the traveltime difference decreases until it reaches the right crossover point of shot gather j and then remains constant. Then, if we calculate the first and second derivatives of the traveltime differences, the crossover point locations correspond to the maximum value of the second derivative. This result is due to a maximum change of slope on the traveltime differences at the first and second layer inflection point and allows automatic crossover point location picking for each overlapping shot pair spread. The use of all possible overlapping shot pairs allows not only the identification of the crossover point (left and/or right) on each shot gather but also a statistically redundant crossover point location on the same shot spread (left or right).

Another important feature of the traveltime difference analysis is the cancellation of traveltime irregularities due to topographic features. However, in the real data case, traveltime irregularities can also be due to errors in the arrival picking and other instrumental factors, so that filtering is essential. A median filter is first applied to the traveltime difference to remove high-frequency variation and keep the general trend of the traveltime difference, especially the step trend at the crossover point. After the first derivative is calculated, a mean filter can be applied to smooth the short wavelength variations due to small steps in the median filtered traveltime difference curve, and keep the long wavelength variation due to major steps associated with the crossover point locations. The mean filter should be used with care and only if the first derivative curve appears noisy. The window length of these filters is determined by the user and should be tested on each dataset. Also, the first and second derivatives use a parameter called differentiation separation length, which can be defined by the user. This parameter determines the number of samples in between each subtraction for the derivation. For example if one is used the differentiation of the traveltime difference curve corresponds to its simple derivative, i.e. the sample interval is one.

$$\Delta t/\Delta x = [t(x + n\Delta s) - t(x)]/n\Delta s \quad (2.6)$$

where x and Δx are respectively the coordinate of the arrival and the coordinate variation; t and Δt are respectively the time of the arrival and the time variation; and Δs is the distance of a sample interval, while n is the differentiation separation length factor.

The window filter length and the differentiation separation length need to be tested on each different dataset to optimize the crossover point autopicking consistency. This will reduce the amount of manual editing of the crossover points.

Finally, the user can specify an offset range in which the crossover points can be picked. This feature should be used only if the user has sufficient knowledge of the data (near-surface geology, previous identification of some crossover points throughout the entire survey) and can confine the crossover points within a certain offset range from the shot locations. It can also be used if third layer arrivals are suspected at a larger offsets, so that the crossover points might be divided between the first and second layer inflections and the second and third layer inflections. In both cases, the offset range limitation will enable consistent crossover point picking and will save a lot of manual editing.

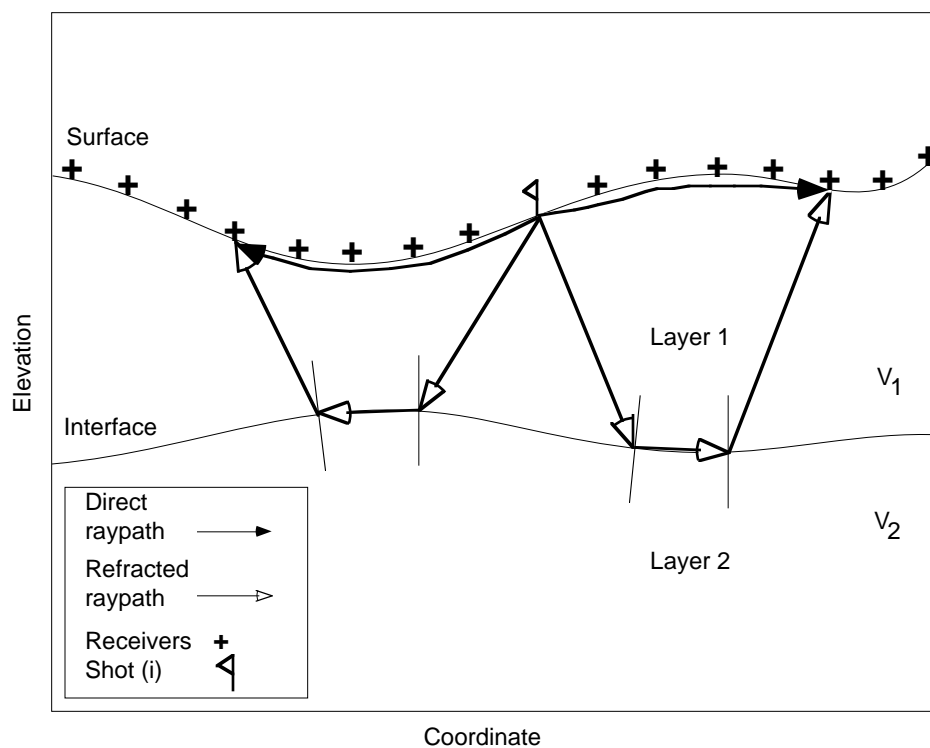


Figure 2.5. Direct and refracted raypath for shot gather *i*.

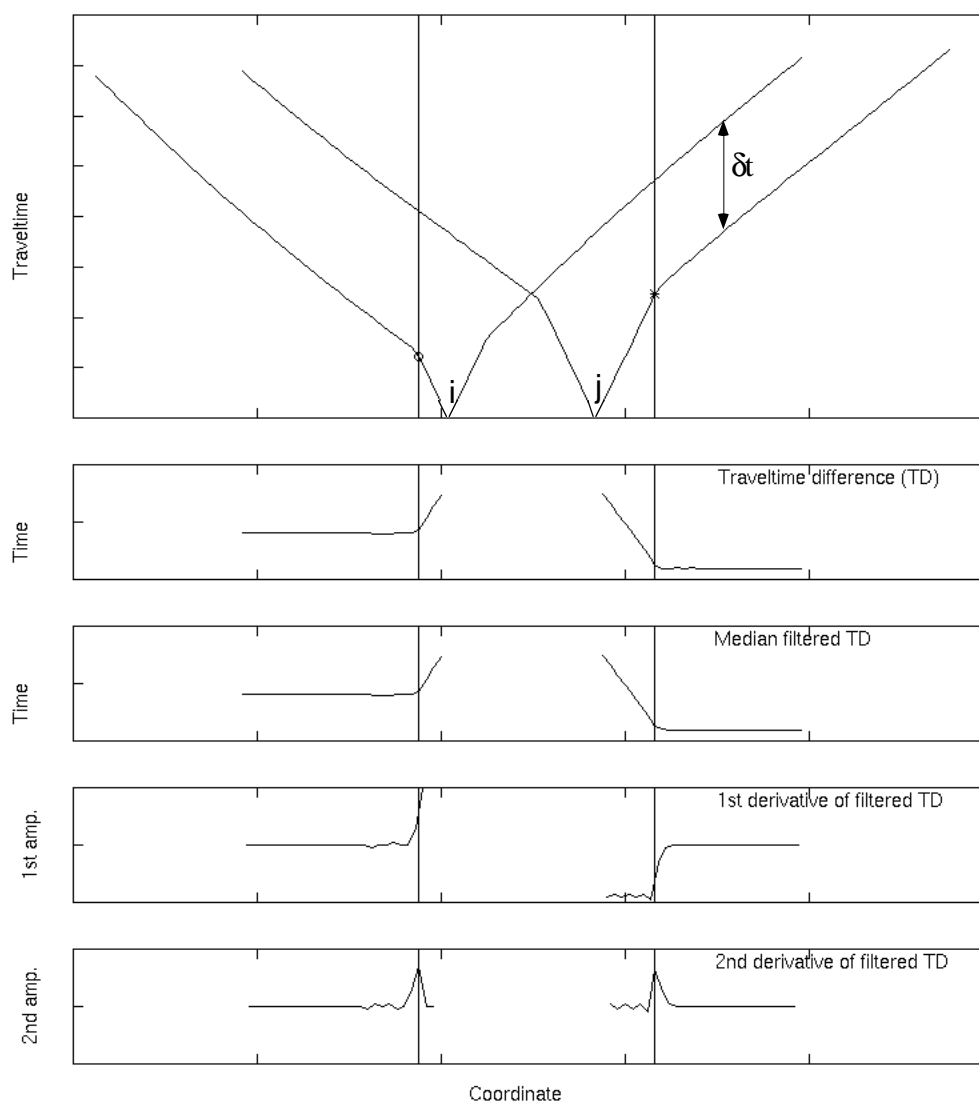


Figure 2.6. Crossover point autopicking of the left crossover point for shot gathers i (o) and of the right crossover point for shot gather j (*), according to the traveltime difference analysis by Lawton (1989) and the branch point analysis of Wang and Cheadle (1995).

2.2.3 Crossover point rejection

This optional process is based on the statistics of the crossover points previously picked. As mentioned in the above section, the use of all possible overlapping shot pairs brings an over determination of the crossover point on a shot spread. The next step is to average all the crossover points that belong to the same shot spread (forward or reverse) and use that value in the subsequent process. The subtraction of each possible overlapping shot spread with a common shot spread should result in consistent traveltime differences and same crossover point surface location. However, if the arrivals are inconsistent and

noisy due picking problem, the traveltimes difference of each overlapping shot spreads might result in different crossover point locations. The utility of the crossover point rejection is to identify and remove the crossover point picks for a given shot spread that have a high deviation from the average crossover point value.

The rejection can be based on a fraction of the standard deviation of the crossover point picks for each shot spread or on a constant deviation limit. The difference between the two rejection limits is that the fraction of the standard deviation limit changes for each crossover point shot spread (right or left) and the constant deviation limit remains constant for all crossover point shot spreads. In both cases, the use of the crossover point statistics, which includes the standard deviation, the fold and the value of the crossover point averages, is recommended to determine the rejection limit. The user should look not only at the average crossover point values, but at the standard deviation and at the fold of each crossover point average. The combination of a high standard deviation with a low fold indicates a crossover point autopicking problem and manual editing might be required, while a low standard deviation with either a high or low fold generally indicates a successful crossover point autopick. In the case of a high standard deviation and a high fold, the crossover point autopicking was noisy, and the crossover point rejection might help remove some bad crossover picks. One thing to keep in mind is that each time crossover point rejection is used, the redundancy (fold) of the crossover point averages is reduced.

2.2.4 Crossover point averaging

The next major step after the crossover point autopicking is the averaging of all the crossover points for a shot spread (left and/or right). For a shot with a leading (right) and a trailing (left) spread, two individual averaging processes are required; one for the left crossover points and one for the right crossover points. In Figure 2.7, the left crossover points of shot gather j comes from the traveltimes difference analysis of its left (trailing) spread with all overlapping left shot spreads to its right, whereas in Figure 2.8, the right crossover points of shot gather j come from the traveltimes analysis of its right (leading) spread with all the overlapping right shot spreads to its left. The proper crossover point positioning is important for the velocity calculation and the Plus time analysis. The crossover points can be edited individually before the crossover point averages are computed. The crossover point averages can also be edited (see PMT method implementation in Appendix C).

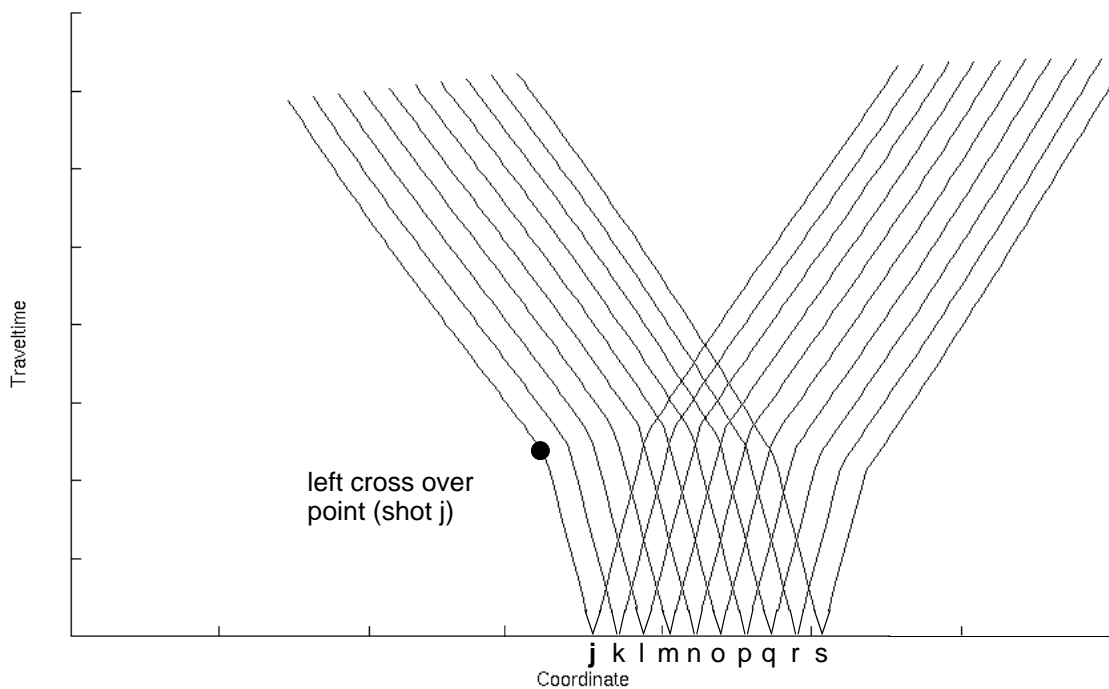


Figure 2.7. Left crossover point determination for shot gather j, established from the overlapping shot spread to the right .

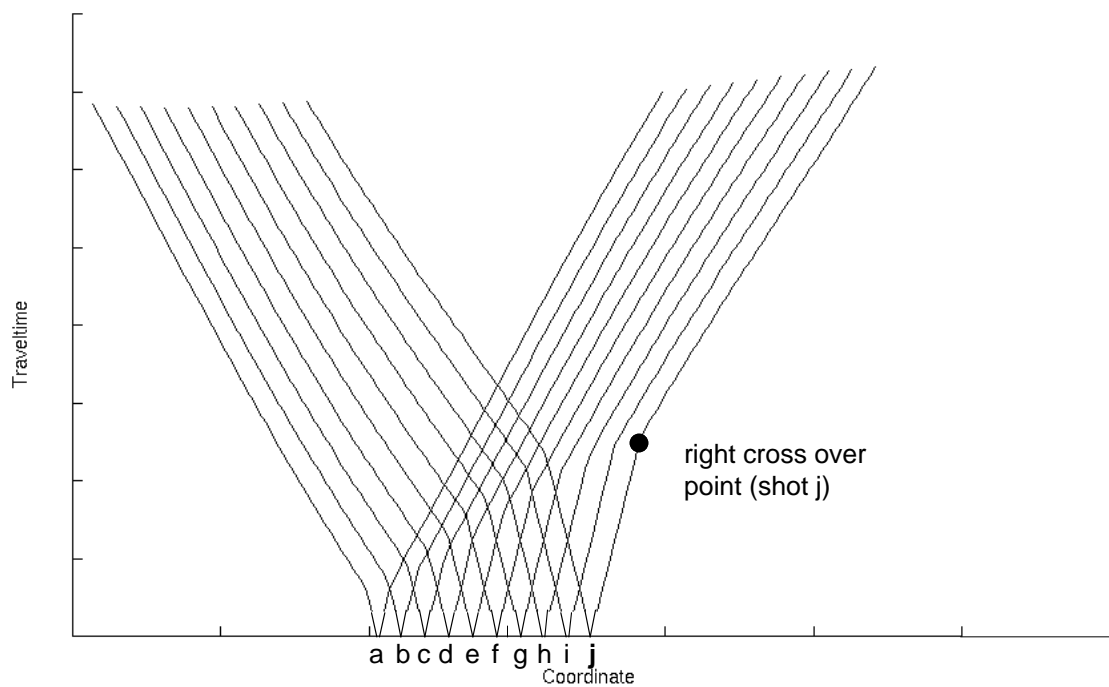


Figure 2.8. Right crossover point determination for shot gather j, established from the overlapping shot spread to the left.

2.2.5 Polyfit and crossover point repicking

This optional process has the objective of obtaining a better crossover point consistency between shot spreads. In fact, the offset distance of the crossover point average of a shot spread from the shot location should not be too different than the offset distance of the crossover point average on the next shot spread from its location. The offset distances between the crossover point averages and their shot locations reveal the general shape of the first layer thickness. Generally, the offset varies proportionally with the variation of the first layer thickness and illustrates its long wavelength shape. If there is an abrupt crossover point average offset change from adjacent shot spreads, it might indicate a crossover point picking problem depending on the distance between their shot locations. The user can solve the problem by editing each crossover point that enters into the crossover point average location or edit directly the crossover point average location (see PMT method implementation in Appendix C). Another way to resolve this problem is to fit a polynomial curve through the crossover point average offset values and repick all the crossover points entering in the average that lie outside a certain deviation from the polynomial curve. Two parameters are defined by the user: the first one is the polynomial degree for the polynomial curve (polyfit curve) and the second one is the deviation limit from the polyfit curve. The polynomial degree determines the amount of complexity in the polyfit curve, i.e. the higher the polynomial degree, the more complex the polyfit curve can be. This parameter should be tested first until the polyfit curve follows the general trend of the crossover point average offsets.

The deviation limit parameter is specified according to the tolerated distance between the crossover point average offsets and the polyfit curve. The deviation limit is determined according to the user interpretation. However, the user should know that the crossover point repicking process will be constrained within an offset range, which is the polyfit curve plus the deviation limit, and the polyfit curve minus the deviation limit. For a small deviation limit, the constraint on crossover point repicking is important, so that the crossover point average offset curve will closely follow the polyfit curve. The user should be concerned about influencing the crossover point repicking process too much, unless he or she has a sufficient knowledge of the data.

2.2.6 Velocity calculation

Once the crossover point average locations are known, then the velocities of the first and second layer can be calculated. A polynomial fit of the direct arrivals will enable the determination of the first layer velocity, while the Minus time analysis of the refracted arrivals will establish the second layer velocity. For a given shot gather, with a leading and a trailing spread, the velocity of the first layer at the shot location is given by the inverse slope of the two direct arrival segments (leading and trailing). The direct arrivals segments are limited by the shot location and the crossover point locations (Figure 2.9). The slope of the direct arrivals is found using a best fit straight line (one degree polynomial fit). Then, the first layer velocity at the shot location corresponds to the average inverse slope of the two direct arrival segments. Finally, the first layer velocity values at each surface station between shotpoints are interpolated.

The second layer velocity is found using Minus time analysis on the refracted arrivals within the Plus-Minus time analysis window (Figure 2.9 and equation 2.5). The Minus time values are calculated for each receiver location inside the Plus-Minus time analysis window. Then, the Minus time variations (ΔT_D) are displayed as a function of twice the receiver separation ($2\Delta x$) (Figure 2.10).

Finally, a straight line is fit through the Minus time variation values, with the inverse slope of it corresponding to the second layer velocity. The second layer velocity value belongs at the half-way location in the Plus-Minus time analysis window, which is defined by the right crossover point of the shot gather i and the left crossoverpoint of the shot gather j . After all the possible Plus-Minus time analysis windows (different shot pairs) have been used, an average of all the second layer velocity values at the same location is calculated, followed by interpolation to the receiver locations. Both first and second layer velocities can then be edited manually and median filtered to remove the high frequency variation and any bad velocity values (see PMT method implementation in Appendix C).

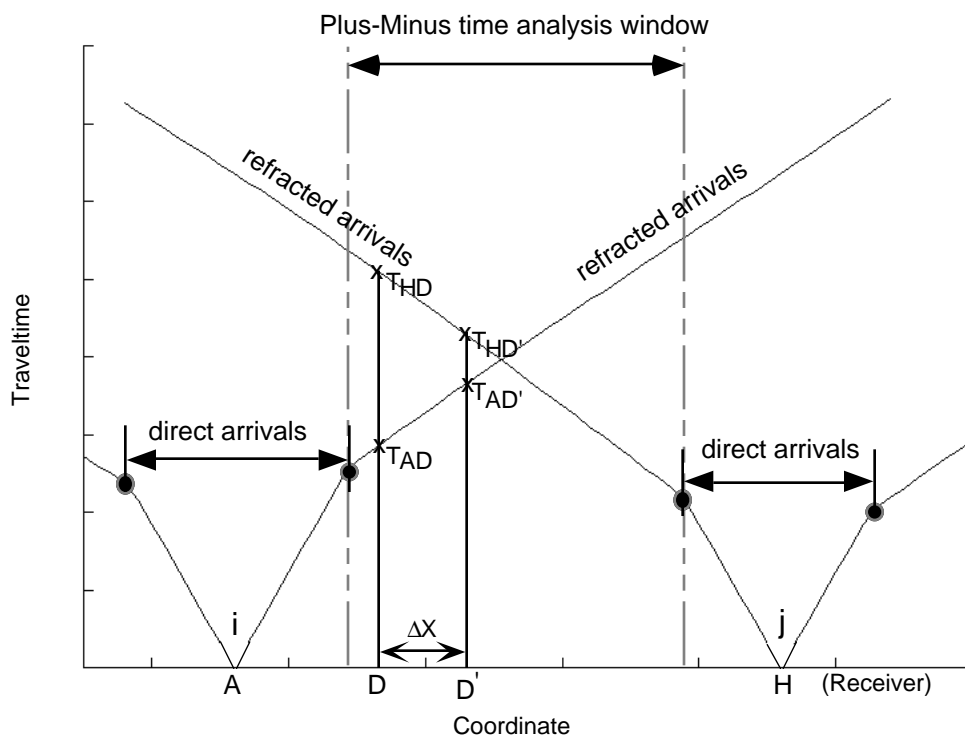


Figure 2.9. Shot gathers i and j with their left and right crossover points, and the Plus-Minus time analysis window .

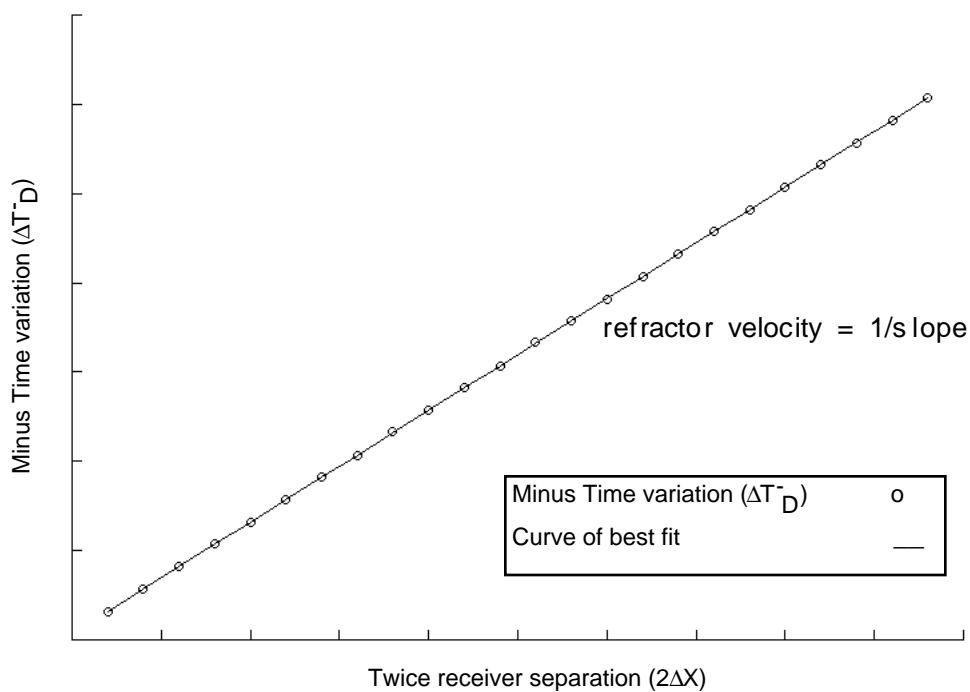


Figure 2.10. Minus time variation displayed in function of twice the receiver separation ($2\Delta x$) and best-fit straight line.

2.2.7 Plus time and Delay time analyses

The Plus time analysis process, like the velocity calculation, requires the determination of the crossover point average locations. As explained in the theory section, the Plus time analysis uses the Delay time analysis concept to calculate the thickness of the first layer below each receiver. This process calculates the Plus time values that are going to be used for the first layer thickness computation in the depth calculation process. The Plus time analysis computes the Plus time values for each receiver within all the possible Plus-Minus time analysis windows (use of different shot pairs). All the Plus time values are stored under the associated receiver locations. The use of all the possible overlapping shot spreads allows a redundant determination of the Plus time values at each receiver location. This is important to reduce the effect of noisy data and bad picks. Some of the Plus time values can be rejected according to their deviation from the average Plus time value at a receiver location (see Plus time rejection section). As mentioned previously, the Plus time analysis process needs the crossover point average locations. Some of the crossover point averages may be missing, because they have been deleted or not computed. In this case, they need to be interpolated from the other crossover point average values or the shot spread without a crossover point average value will not be used in the Plus time analysis process. Also, if the presence of third layer arrivals is known, these arrivals can be excluded from the Plus time analysis computation by simply limiting the offset range.

The Plus time analysis cannot be used to determine the Plus time values for the receivers outside the Plus-Minus time analysis window limit (Figure 2.11). Therefore, the Delay time analysis is computed for these receivers (Figure 2.12) (Lawton, 1989). The Delay time value at a receiver can be expressed as:

$$\delta_r = T_r - \delta_s - X/V_2 \quad (2.7)$$

where T_r is the refracted travelttime at the receiver, δ_s is the shot Delay time, X is the distance between the shot and the receiver, and V_2 is the second layer velocity.

The shot Delay time (δ_s) is equivalent to half the Plus time value at that surface location and the receiver delay time can be multiplied by two to obtain the Plus time value at this surface location. So, the Plus time analysis has to be computed prior to the use of the Delay time analysis. Also, the second layer velocity has to be already calculated or known. The use of the Delay time analysis in conjunction of the Plus time analysis allows the determination of the Plus time values at each surface location.

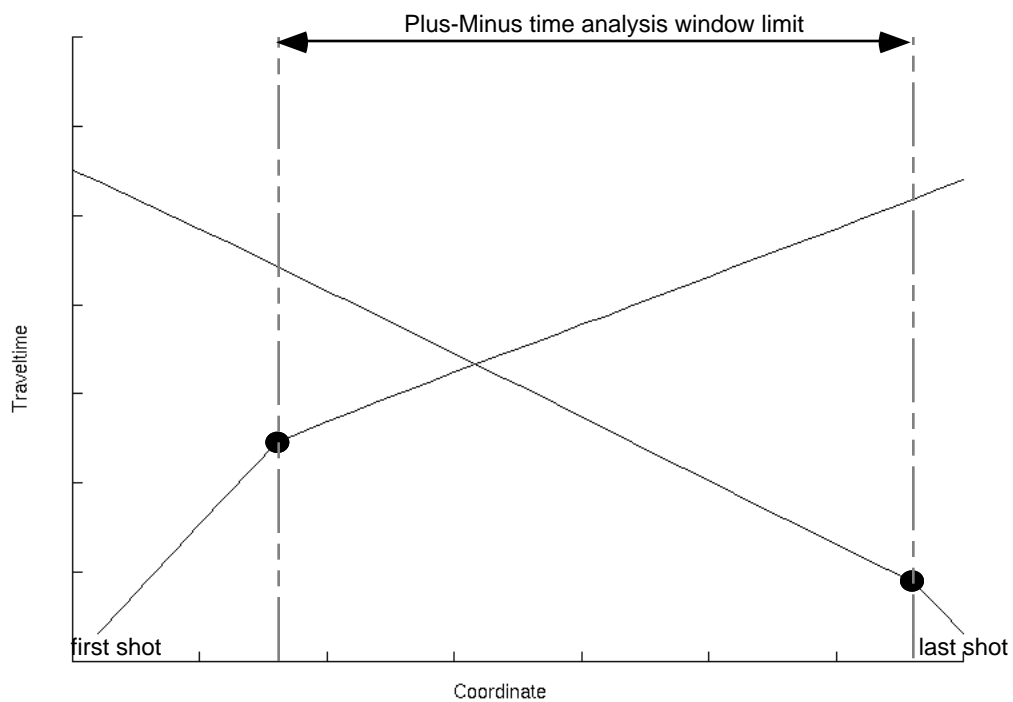


Figure 2.11. Plus time analysis limitation.

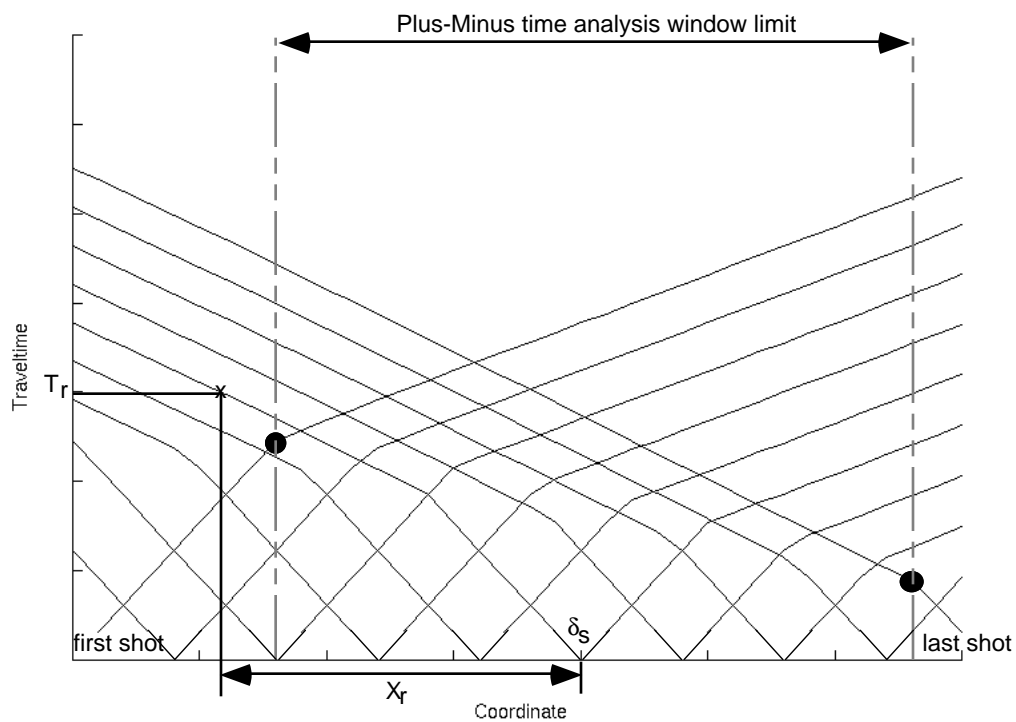


Figure 2.12. Delay time analysis for receiver locations outside the Plus-Minus time analysis window limit.

2.2.8 Plus time rejection

The Plus time rejection process is similar to the crossover point rejection in terms of the statistical rejection concepts. The rejection can be done according to a constant deviation limit or according to a fraction of the standard deviation limit. The constant deviation limit remains unchanged for all the receiver locations, while the standard deviation changes for each receiver location. At each receiver locations, the Plus time average value, the standard deviation and the fold are known and can be evaluated. The Plus time average curve should have a reasonable long wavelength trend combined with short wavelength variations. If a Plus time average value is completely out of the range of the other Plus time average values and the standard deviation for this Plus time average value is high, it reveals that there are some bad Plus time values entering into the Plus time averaging at this receiver location. In general, high standard deviation indicates the presence of bad Plus time values, which can be rejected using the Plus time rejection process. Again, like the crossover point rejection, the Plus time rejection process will reduce the redundancy (fold) of the Plus Time averages. The Plus time rejection process should be used before the Delay time analysis computation.

2.2.9 Depth calculation

After the Plus time analysis is completed, the Plus Time average values at each receiver are calculated. At this stage, the first and second layer velocity at each receiver, as well as the Plus time average, should be known. The calculation of the first layer thickness below each receiver is based on the delay time analysis concept (Equation 2.2). Before the depths are calculated, the user has to make sure that the Plus time average values are consistent and that the two layer velocities are reasonable, i.e. no noisy short wavelength variations and the first layer velocity is not greater than the second layer velocity. The Plus time average values, and the first and second layer velocities are needed to evaluate the thickness of the first layer below each receiver (Figure 2.13 and Equation 2.8). The first layer thickness can be manually edited or median filtered before the static corrections are computed to reduce the high frequency noise (see PMT method implementation in Appendix C).

$$Z_1 = [(T^+) * (V_1)] / 2(\cos(\theta_c)) \quad (2.8)$$

$$\text{where } \theta_c = \sin^{-1}(V_1/V_2)$$

2.2.10 Static computation

The final process computes the static corrections to be applied to the seismic data. The corrections are for the weathering layer (first layer) and for the surface elevation variation (topography). The weathering correction will cancel the time delays produced by the first low velocity layer. The elevation correction will bring the shots and the receivers to a common flat datum. The static computation process is based on the depth and velocity model and will calculate the receiver static corrections and shot static corrections based on surface-consistent static corrections.

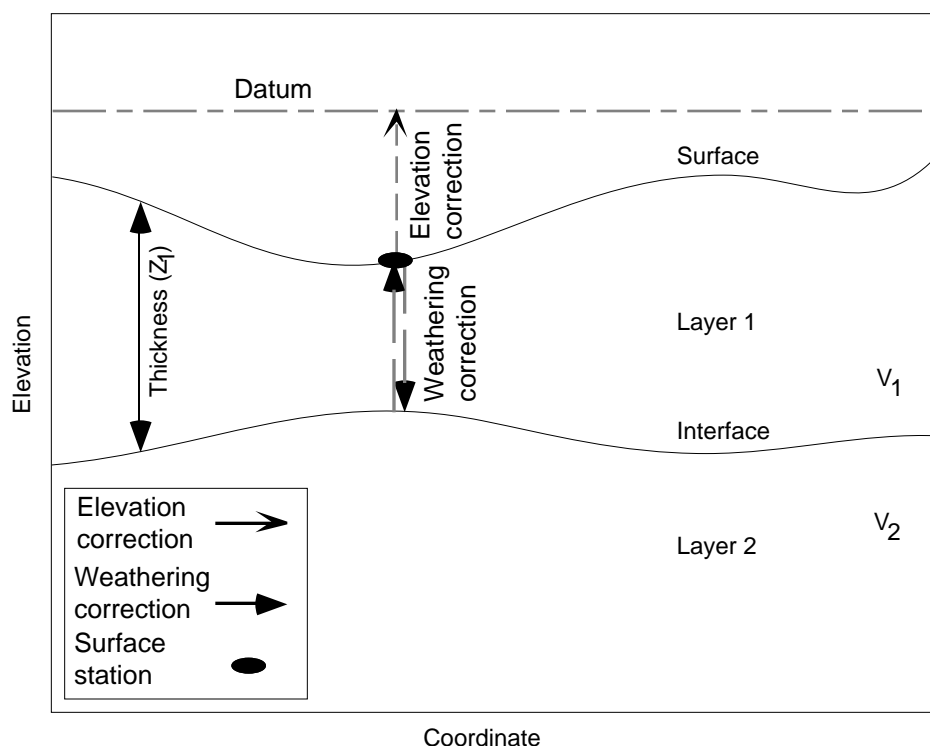


Figure 2.13. Surface-consistent static corrections including elevation and weathering static corrections for a 2 layer model.

Weathering correction

The weathering correction corresponds to a time correction which essentially replaces the weathering layer velocity with the second layer velocity (Figure 2.13). If the shot is buried below the surface, the uphole time is used to bring the shot back to the surface (before the weathering correction is applied). Then, the shot or the receiver is brought back to the surface using the replacement velocity. The replacement velocity is normally chosen to be the second layer velocity, either the local or average value.

The weathering static correction (Δt_w), for a shot or a receiver can be expressed as follows:

$$\Delta t_w = -Z_1/V_1 + Z_1/V_{rep} + T_{uh} \quad (2.9)$$

where Z_1 is the first layer thickness, V_1 is the first layer velocity, V_{rep} is the replacement velocity used, and T_{uh} is the uphole time.

Elevation correction

The elevation correction (Δt_e) corresponds to the time needed to bring back the receiver or the shot from the surface to a flat datum using the replacement velocity (both flat datum and replacement velocity are user defined).

$$\Delta t_e = (H_d - H_s)/V_{rep} \quad (2.10)$$

where H_d is the datum elevation, and H_s is the surface elevation.

Total correction

Finally, the total correction (Δt) is a summation of the weathering and elevation static corrections. The total static correction for a shot or a receiver can be expressed as follow:

$$\Delta t = \Delta t_w + \Delta t_e \quad (2.11)$$

$$\Delta t = -Z_1/V_1 + Z_1/V_{rep} + (H_d - H_s)/V_{rep} \quad (2.12)$$